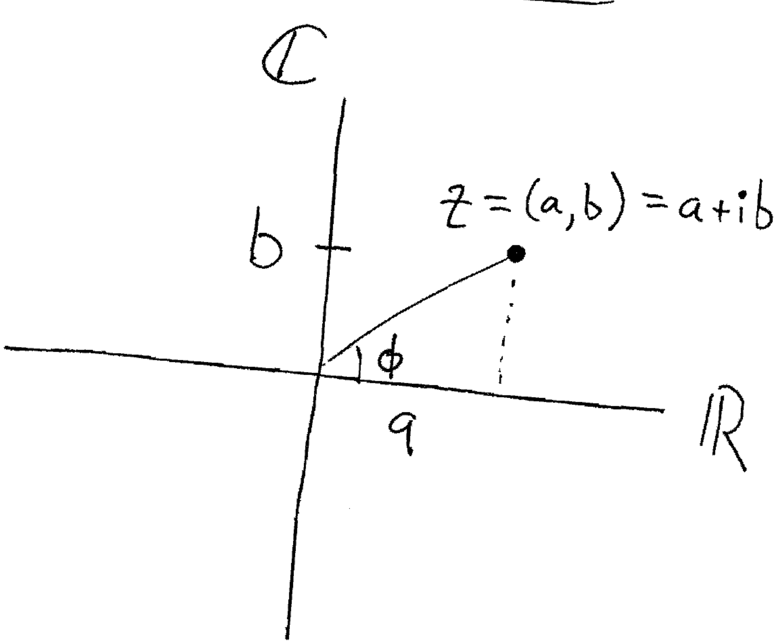
Complex Variable Basics:

A complex number z is written as,

$$\boxed{z = a + ib} \quad \text{where } i = \sqrt{-1}$$

a is the real part of z, and b is the imaginary part of z. It is important to note that both a and b are real numbers.

Complex Plane:

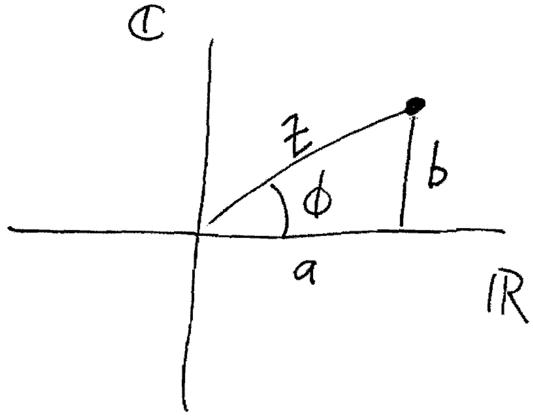


The convention is to plot the real axis horizontally, and the complex axis vertically.

Polar Form:

$$\boxed{z = a + ib = |z| e^{i\phi}}$$

Proof:



$$|z| = \sqrt{a^2 + b^2}$$

$$\cos\phi = \frac{a}{\sqrt{a^2+b^2}}$$

$$\sin\phi = \frac{b}{\sqrt{a^2+b^2}}$$

$$\underbrace{\cos\phi + i\sin\phi}_{= e^{i\phi}} = \frac{a+ib}{\sqrt{a^2+b^2}} \quad \rightarrow = |z|$$

$$\rightarrow \quad \boxed{|z| e^{i\phi} = a + ib}$$

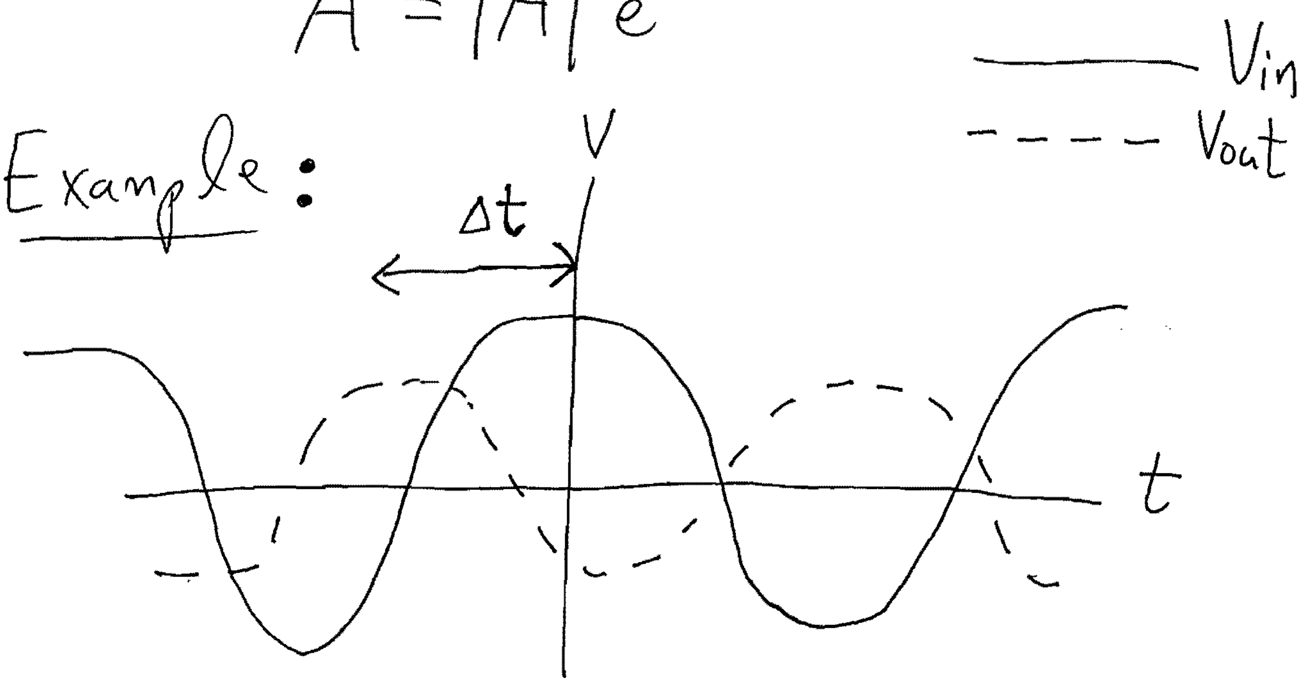
where $\boxed{\phi = \tan^{-1}\left(\frac{b}{a}\right)}$

Why use polar form?

As you'll see we use response functions to explain the frequency dynamics of circuits. Response functions are complex:

$$A = |A| e^{i\phi}$$

Example:



Given some input signal which interacts with a ciruit with response function $A = \frac{V_{out}}{V_{in}}$,

$$\boxed{|V_{out}| = |A|\, V_{in}}$$

In other words the magnitude of a response function tells you about the amplitude of your output.

$$\boxed{\phi = \frac{2\pi \Delta t}{T}}$$

When $A = a + ib$ recall that $\phi = \tan^{-1}\left(\frac{b}{a}\right)$, this angle ϕ tells you how much horizontal phase shift your output signal will have.